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REFLECTION OF A DAM-BREAK WAVE AT A VERTICAL WALL. NUMERICAL MODELING AND EXPERIMENT

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Results of the experimental study and numerical modeling of the reflection of a dam-break wave at the vertical end wall of a channel are given. A wave forms with distance from a partition creating the initial level difference of the liquid. It is shown that a numerical calculation based on the Zheleznyak–Pelinovskii nonlinear dispersion model satisfactorily describes the height of the splash-up, the amplitude of reflected waves, and the wave velocity in front of the wall for smooth and dam-break waves. It is also shown that, for smooth and weakly breaking (without significant entrainment of air) incoming waves, the experimental values of the height of the splash-up at the wall agree well with relevant experimental and calculated data for solitary waves.

Formulation of the Problem. The problem of the formation of a dam-break wave and its subsequent reflection at a wall is considered. The system consists of a reservoir with an even bottom and rigid impenetrable vertical walls. The reservoir is filled with a homogeneous liquid and has a thin impenetrable partition creating the level difference of the liquid (Fig. 1) at the initial moment. This formulation includes two classical problems: the problem of the formation of a dam-break wave and the problem of wave reflection at a wall. In the present work, the processes that occur in a part of the reservoir with the initially lower level of the liquid were investigated.

Many studies deal with dam-break waves that arise, for example, in breaking a dam. Previously, their parameters were calculated with the use of the Saint-Venant equations which do not take into account the dispersion. In this case, breaking waves are modelled by discontinuous solutions. The Saint-Venant equations do not describe a broad class of problems (smooth undular and solitary waves). At the same time, experiments [1] have shown that in a certain range of parameters $\Delta h^0 = (h_1 - h_2)/h_2$ and $l^0 = l_2/h_2$, the dam-break wave is accompanied by oscillations behind the smooth front, i.e., it has the form of a smooth undular bore.

It follows from theoretical results [2] that the energy lost by a breaking bore is partially expended on the formation of oscillations in the form of cnoidal waves. Calculations with the use of models that allow for the dispersion were performed, in particular, in [3, 4].

For the considered case of the reflections of a dam-break wave, the Saint-Venant equations give quite an exact value of the splash-up height only for small-amplitude waves. For a particular reservoir, the numerical modeling of the reflection of a dam-break wave is described in [5]; note that the calculations were performed with the use of the Zheleznyak–Pelinovskii nonlinear dispersion model and the model of potential fluid motion. Numerous studies are devoted to the reflection of solitary waves (see, e.g., [6–9]). Below, it is shown that the calculation results obtained for the reflection of a solitary wave at a vertical wall on the basis of the equations of potential fluid motion are in good agreement with those obtained for the reflection of a smooth dam-break wave.

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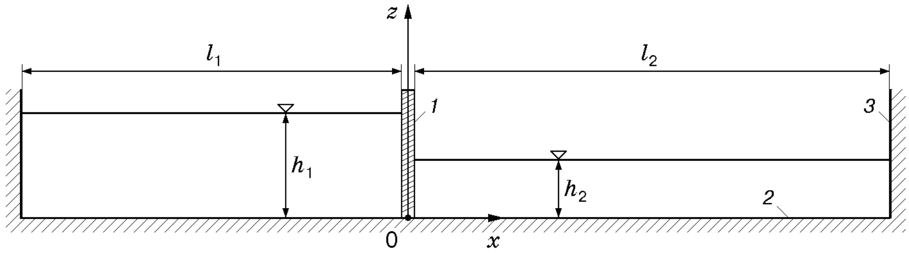

Fig. 1. Layout of the setup: 1) partition; 2) bottom of the channel; 3) reflecting wall.

Experimental Technique. The experiments were carried out in a closed rectangular Plexiglas channel of width 6 cm with an even horizontal bottom. The waves were generated by removing a partition creating the initial level difference $\Delta h = h_1 - h_2$. In addition to h_1 and h_2, the important parameter of the problem is the distance between the partition and the reflecting wall l_2. The linear dimensions were normalized by the value of h_2, and the velocities by the value of $\sqrt{gh_2}$ (g is the acceleration of gravity).

The experimental technique is similar to that given in [10]. A fine disperse powder was preliminarily coated on the end wall; a certain portion of this powder was carried away by the incoming wave, producing a distinct boundary of the splash-up, i.e., the maximum rise of the liquid level, at the wall. The structure and velocity of the wave were recorded by a wavemeter and video recorded. The resistive wavemeter consisted of two electrodes of diameter 0.1 mm. The recording equipment included a self-recorder or an analog-to-digital converter connected to a computer. Video recording was performed by a Panasonic RX2 VHS-C video camera with a frequency of 25 frames/sec. The low resolution of the video camera (240 lines in the vertical) was compensated for by recording with large magnification, in which the leading crest of the wave occupied approximately half of the frame. The camera was moved on a tray in such a way that the first crest of the wave remained in the frame. To obtain the most contrast image, the sports mode of video recording with an exposure time of approximately 1/500 sec was used. Then, the image was computer-processed. The measurement error for the height and velocity of the leading crest, which is caused by smearing of the wavefront image, was 3%. The partition was removed by hand. The time for which it was removed from the liquid was approximately 0.1 sec; as the pictures of the internal structure show, the perturbations caused by adhesion of the liquid to the partition either rapidly disappeared for $\Delta h^0 < 0.8$ or transformed to a breaking hydraulic jump. After the partition was removed, a wave with elevation of the level was propagated over the smaller-depth liquid and a wave with lowering of the level was propagated over the larger-depth liquid. In this work, the wave with elevation of the level was investigated. During evolution of the dam-break wave the height of the first crest a_1^0 and the propagation velocity of the middle point of the leading edge c^0 (hereinafter, the superscript 0 corresponds to dimensionless quantities) were measured. The problem parameters were varied in the following ranges: $0.5 \leqslant \Delta h^0 \leqslant 1.4$, $50 < l_2/h_2 < 90$, and $h_2 = 3$–4 cm.

Numerical Model. The formation of an undular bore is due to dispersion effects; therefore, the Zheleznyak–Pelinovskii nonlinear dispersion model was used in calculations [11]. In dimensionless variables, the equations of this model are written in the form

$$h_t + P_x = 0, \qquad P_t + \Big(\frac{P^2}{h} + \frac{h^2}{2}\Big)_x = D. \tag{1}$$

Here $D = ((h^3/3)(u_{xt} + uu_{xx} - (u_x)^2))_x$ in the case of a horizontal bottom of the reservoir. Here h is the total depth of the liquid, u is the velocity of the liquid, and $P = hu$ is the flow rate of the liquid.

Similarly to the technique described in [12], the system of Zheleznyak–Pelinovskii equations was split in the second-order ordinary differential equation

$$(h^3 d_x)_x - 3hd = (h^3(2(u_x)^2 + h_{xx}))_x \tag{2}$$

relative to the unknown d and a hyperbolic system of the form (1), where $D = hd$.

The calculations were performed on a mobile adaptive grid. Equation (2) was first solved; then, the hyperbolic system was approximated with the use of the TVD-scheme described in [13].

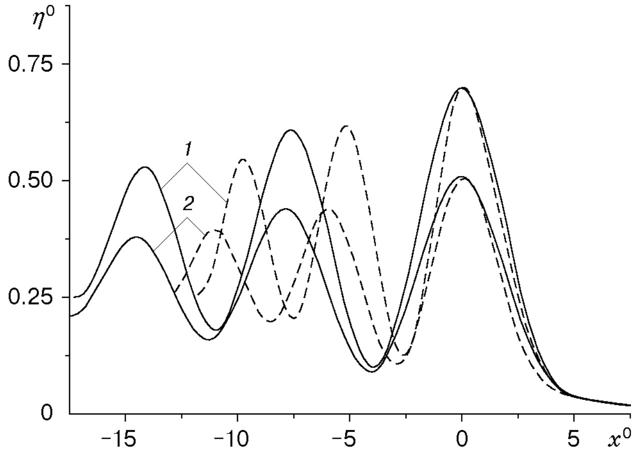

Fig. 2. Comparison of the experimental and calculated profiles of the dam-break waves for $\Delta h^0 = 0.8$ (curves 1) and 0.6 (curves 2); solid and dashed curves refer to the calculated and experimental profiles, respectively.

Experimental Results and Analysis. As is known, breaking of nonstationary waves of many types occurs when a certain critical velocity of the leading-edge particles is reached [14, 15]. This velocity is called the second critical velocity and it is equal to the limiting velocity of solitary waves $c^0_{\rm cr} = 1.294$ obtained in [16] by numerical calculations based on the complete equations of potential fluid motion. The experimental results reported here show that the dam-break waves begin to break precisely at this velocity. If the velocity of the leading edge of the dam-break wave did not exceed $c^0_{\rm cr}$ during its evolution, the wave remained smooth. This behavior was observed for $\Delta h^0 < 0.8$. For $0.8 < \Delta h^0 < 1.1$, both smooth stages and stages with leading-edge breaking were observed in the evolution of the wave. For $\Delta h^0 > 1.1$, breaking of the leading edge occurred up to the moment of interaction with the wall. It is noteworthy what even the incoming waves with the breaking leading edge became smooth after reflection and they could break again after a time.

Figure 2 shows experimental (dashed curves) and calculated (solid curves) profiles of the dam-break waves. The profiles are obtained for $l^0_2 = 61.5$; note that the pair of profiles with the larger value of elevation $\eta^0 = h^0 - 1$ corresponds to the case $\Delta h^0 = 0.8$, and with the smaller value to the case $\Delta h^0 = 0.6$. The curves are arranged in such a way that the peaks of the first undulations have the same abscissa.

It follows from Fig. 2 that the undulations, their height, and the depth of troughs between undulations almost coincide. A certain difference is observed in the undulation width: the calculated profiles are somewhat wider than the experimental ones. A similar result was observed earlier in comparing the profile of a solitary wave (solutions of the Corteweg-de-Vries equation) with experiment in [17], where the solitary-wave profiles obtained with the use of simple models are compared in detail.

Figure 3 shows the characteristic experimental profiles of the dimensionless splash-up ζ^0, i.e., the maximum level of the liquid at the wall in the process of wave reflection (the boundary dividing the wetted and nonwetted regions of the wall surface after reflection of the wave). The dimensionless transverse coordinate on the wall y^0_* is plotted on the abscissa. Profiles 1 and 2 obtained upon reflection of smooth waves are uniform in height. Profiles 3 and 4, which correspond to incoming dam-break waves, have strong oscillations. For an analysis of the profiles, we introduce three characteristic values of ζ^0: maximum ($\zeta^0_{\rm max}$), minimum ($\zeta^0_{\rm min}$), and channel width-averaged ($\overline{\zeta^0}$) values. In our numerical study, the plane problem was considered, i.e., the transverse profile of the splash-up was assumed to be horizontal. Therefore, the averaged value of the experimental profile of the splash-up was compared with the numerical one.

The dependence of the splash-up height on the level difference is given in Fig. 4. The break is clearly seen on the diagrams. For $\Delta h^0 \approx 1.05$, a significant decrease in the rate of increase in the splash-up height with increase in Δh^0 is observed. In addition, the difference between $\zeta^0_{\rm max}$ and $\zeta^0_{\rm min}$ substantially increases. Visual observations and video recording have shown that, for $\Delta h^0 > 1$, the incoming waves become breaking waves near the wall. Thus, the part of the curve to the left of the break corresponds to the splash-up of smooth waves and that to the right corresponds to the splash-up of the dam-break waves.

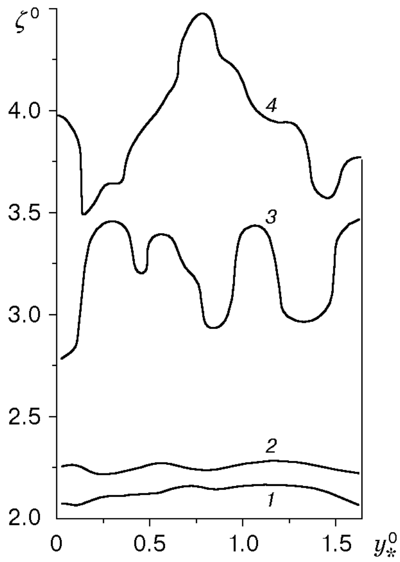

Fig. 3. Transverse profiles of the splash-up: curves 1 and 2 refer to smooth waves for $\Delta h^0 = 0.74$ (1) and 0.89 (2) and curves 3 and 4 refer to breaking waves for $\Delta h^0 = 1.14$ (3) and 1.46 (4).

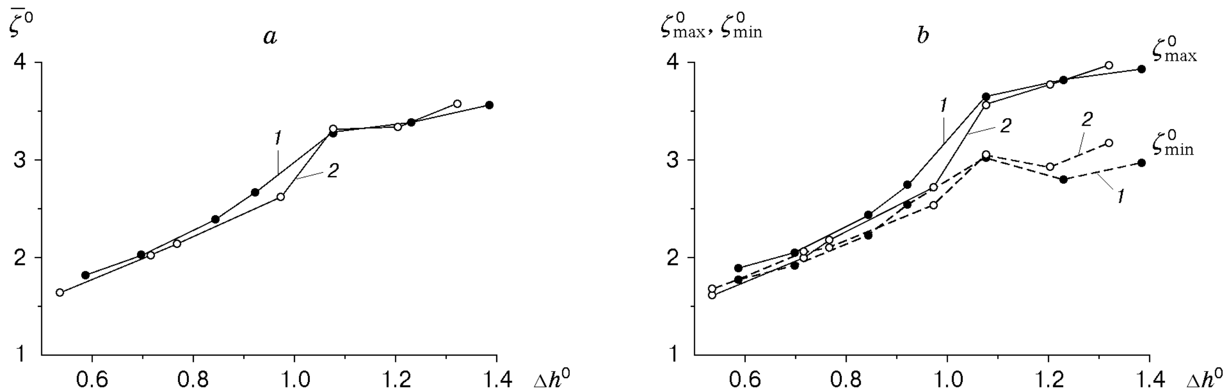

Fig. 4. Dependence of the average (a) and maximum and minimum (b) heights of the splash-up of the dam-break wave on the level difference $l_2/h_2 = 71.8$ (curves 1) and $l_2/h_2 = 61.5$ (curves 2).

Figure 5 shows experimental and calculated dependences of the splash-up height and the amplitude of the reflected solitary waves on the level difference. The amplitudes of the reflected waves were measured at the moment at which their interaction with the incoming waves terminated; note that in calculations, the later moment of time corresponded to the larger values of l_2^0, which caused a certain nonmonotone character of the dependence of the amplitude of the reflected wave on l_2^0.

The experimental and calculated values of the splash-up height of the reflected waves and their amplitudes correlate satisfactorily. For the numerically determined height of the splash-up, one curve is given, because the curves obtained for the three values of the parameter l_2/h_2 coincided with an accuracy up to 0.5%. The experimental data differ from the calculation results by no more than 10%. This discrepancy is the same for all the series of tests with level differences corresponding to the conditions of the formation of smooth waves. For modes with breaking waves, for $\Delta h^0 > 1$, the experimental data are in a better agreement with calculations, especially in the case $l_2/h_2 = 51.3$.

The calculated values of the amplitudes of the reflected waves coincide at small level differences. For the largest level differences, the discrepancy between the calculated values does not exceed 6%. The experimental curves lie below the calculated ones; at large level differences, they are different by no more than 6%. For

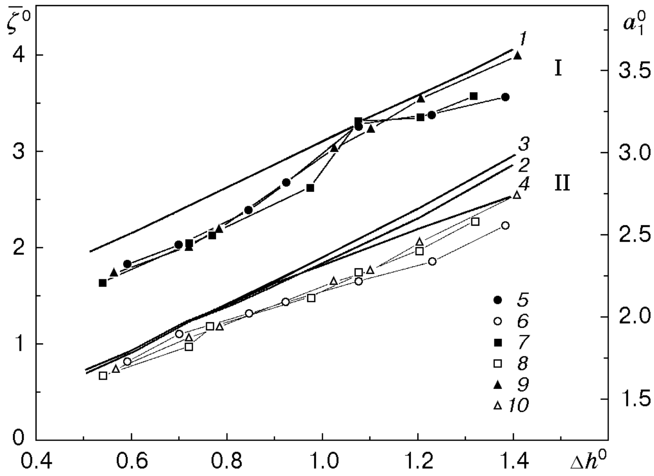

Fig. 5. Dependence of the splash-up height (I) and the reflected-wave amplitude (II) on the level difference: solid curves refer to calculation and points refer to experiment; curve 1 refers to $l_2^0 = 51.3$, 61.5, and 71.8 (three curves coincided with an accuracy up to 0.5%), curve 2 and points 5 (I) and 6 (II) refer to $l_2^0 = 51.3$, curve 3 and points 7 and 8 refer to $l_2^0 = 61.5$, and curve 4 and points 9 and 10 refer to $l_2^0 = 71.8$.

l_2/h_2, the difference between the calculated and experimental values of the amplitude of the reflected waves does not exceed 8%.

It follows from a comparison of the measured and calculated wave velocities that the experimental values are approximately 5% smaller than the calculated ones in the range $0.5 \leqslant \Delta h^0 \leqslant 1.4$. Because the wave propagation was nonstationary and it was characterized by different velocities of the leading-edge points (both for smooth and breaking waves), the velocity of motion of the average-in-height point of the leading edge was used as a characteristic wave velocity.

As has already been mentioned, as the second critical wave velocity was reached, breaking of the leading edge was observed in the experiment in the direction of the wave propagation, and the propagation velocity of the broken front could increase. In numerical calculations, the process of breaking with increase in the velocity and amplitude was not observed.

It was revealed in experiments [10] that if the incoming dam-break wave is smooth, the height of its splash-up correlates well with the splash-up height of solitary waves. We analyze this problem in more detail. The magnitude of the maximum splash-up of a solitary wave at a vertical wall was investigated in [6–9]. In [9], the following formulas containing terms from the first to the third order are obtained with the use of the perturbation method applied to the equations of potential fluid motion:

$$\overline{\zeta^0} = 2a^0; \tag{3}$$

$$\overline{\zeta^0} = 2a^0 + (a^0)^2/2; \tag{4}$$

$$\overline{\zeta^0} = 2a^0 + (a^0)^2/2 + 3(a^0)^3/4. \tag{5}$$

Here a^0 is the dimensionless amplitude of a solitary wave. In [6, 7], the splash-up height of a solitary wave is calculated numerically on the basis of the complete equations of potential motion. In [8], five types of splash-up of solitary waves at a wall are identified experimentally. It is noteworthy that the dependence of the splash-up height on the amplitude of the incoming wave in the form of a soliton by means of the Zheleznyak–Pelinovskii model is expressed by formula (4).

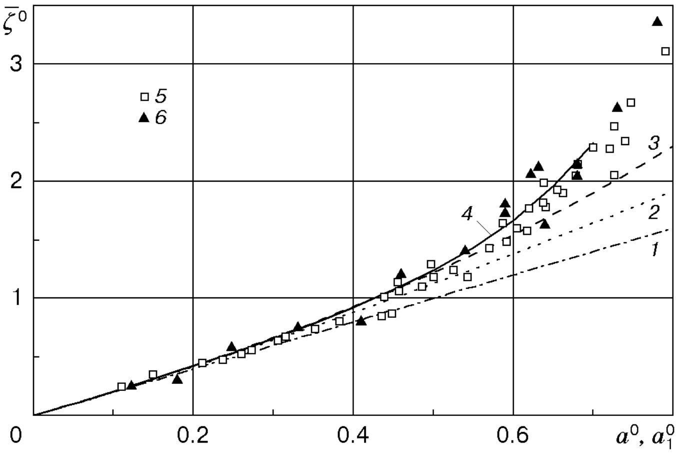

Fig. 6. Dependence of the splash-up height of solitary (curves 1–5) and dam-break (curve 6) waves on the amplitude: curves 1–3 refer to calculations by formulas (3), (4), and (5), respectively, curve 4 refers to calculation by means of the model [6]; points 5 and 6 refer to experimental results of [8] and results of the present experiments, respectively.

Figure 6 shows the dependence of the splash-up height on the amplitude for solitary and dam-break waves. Experimental points 5 from [8] and points 6 obtained in the present experiments are in satisfactory agreement, although they are obtained in channels of different dimensions and by different methods for waves of two types. In the case of solitary waves, the experiments are in better agreement with the calculation performed according to the model of potential flows [6] (curves 4) and the calculation in [7]. Dependence (5) (curve 3) corresponds well to experiments up to the height of the incoming dam-break wave $a_1^0 < 0.55$. Formulas (3) and (4) (curves 1 and 2, respectively) describe well the process only for $a_1^0 < 0.4$. We note that the calculations in [6] were performed only for $a^0 \leqslant 0.7$.

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REFERENCES

1. H. Favre, *Ondes de Translation Dans les Canaux Decoverts*, Dunod, Paris (1935).
2. T. B. Benjamin and M. J. Lighthil, "On cnoidal waves and bores," *Proc. Roy. Soc., London, Ser. A*, **224**, 448–460 (1954).
3. D. H. Peregrine, "Calculations of the development of an undular bore," *J. Fluid Mech.*, **25**, Part 2, 321–330 (1966).
4. P. K. Stansby, A. Chegini, and T. C. D. Barnes, "The initial stages of dam-break flow," *J. Fluid Mech.*, **374**, 407–424 (1998).
5. V. B. Barakhnin, G. S. Khakimzyanov, L. B. Chubarov, and D. A. Shkuropatskii, "Some problems of numerical modeling of wave modes in enclosed water areas," *Vychisl. Tekhnol.*, **1**, No. 2, 3–25 (1996).
6. M. J. Cooker, P. D. Weidman, and D. S. Bale, "Reflection of high-amplitude solitary wave at a vertical wall," *J. Fluid Mech.*, **342**, 141–158 (1997).
7. B. E. Protopopov, "Numerical analysis of the transformation of a solitary wave upon reflection at a vertical barrier," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 115–123 (1990).
8. J. P. McHugh and D. W. Watt, "Surface waves impinging on a vertical wall," *Phys. Fluids*, **10**, No. 1, 324–326 (1998).
9. C. H. Su and R. M. Mirie, "On head-on collisions between two solitary waves," *J. Fluid Mech.*, **98**, 509–525 (1980).

10. V. I. Bukreev and A. V. Gusev, "Reflection of a dam-break wave at a vertical wall," in: *Trans. of the Novosibirsk State Architecture and Building Institute* [in Russian], Vol. 3, No. 2, 47–59 (2000).
11. M. I. Zheleznyak and E. N. Pelinovskii, "Physicomathematical models of tsunami running at the shore," in: *Institute of Applied Physics* [in Russian], Inst. of Appl. Phys., Acad. of Sci of the USSR, Gor'kii (1985), pp. 8–33.
12. V. B. Barakhnin and G. S. Khakimzyanov, "On the application of adaptive grids to the numerical solution of one-dimensional problems in the shallow-water theory," *Russ. J. Numer. Anal. Math. Modeling*, **10**, No. 5, 373–39 (1995).
13. V. B. Barakhnin and N. B. Borodkin, "TVD-scheme of the second order of approximation on a mobile adaptive grid for hyperbolic systems," *Sib. Zh. Vychisl. Mat.*, **3**, No. 2, 109–121 (2000).
14. V. I. Bukreev and A. V. Gusev, "Waves in a channel ahead of a vertical plate," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, **1**, 82–90 (1999).
15. V. I. Bukreev and A. V. Gusev, "Some results of experiments with undular waves," in: *Trans. of the Novosibirsk State Architecture–Building Univ.* [in Russian], Vol. 2, No. 2, 13–22 (1999).
16. M. S. Longuet-Higgins and J. D. Fenton, "On the mass, momentum, energy and circulation of a solitary wave. 2," in: *Proc. Roy. Soc. London, Ser. A*, **340**, 471–493 (1974).
17. J. W. Daily and S. C. Stephan, "The solitary wave. Its celerity, profile, internal velocities and amplitude attenuation in a horizontal smooth channel," in: *Proc. of the 3rd Conf. Coastal Eng.*, Univ. of California Berkley (1952), pp. 13–30.